Optimum Yaw Motion for Satellites with a Nadir-Pointing Payload

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This paper treats the case of a satellite powered by a plane solar array with a payload that needs to be pointed toward nadir at all times. We first derive the formulae for perfect alignment between solar generator and sun direction. These attitude conditions have been derived before by other authors, but the coordinates chosen here permit a particularly simple presentation of the results. A simplified attitude scheme was then proposed which consisted of replacing the ideal yaw angle profile by a square wave. The yaw angle is thus maintained constant for large parts of the orbit, which is a great advantage for certain applications.

Nomenclature

$E(k)$ = elliptic integral of modulus k
G = solar array vector, unit vector normal to the solar array plane (suffixes denote the coordinate system in which the vector is written)
i_p = inclination of the planet orbit to the ecliptic (see Fig. 3)
i_Q = inclination of the satellite orbit to the ecliptic (see Fig. 3)
i_s = inclination of the satellite orbit to the planet orbit (see Fig. 3)
P = $P(\lambda_s,\phi_s)$ = power output of the solar array for arbitrary attitudes
P_m = average value of P
P_0 = maximum value of P, obtained for perfect alignment of the array
S = solar vector, unit vector from satellite to sun (see Fig. 2) (suffixes denote the coordinate system in which the vector is written)
s = right ascension of the sun vector in the X_E-Y_E plane, i.e., angular distance from equinox to S (see Fig. 2)
α = $\omega_Q+\phi_Q$, angular distance X_Q to X_H (see Fig. 2)
ϕ_Q = true anomaly of the satellite (see Fig. 2)
ϕ_s = azimuth of the sun in H coordinates (see Fig. 2)
ϵ_1,ϵ_2 = eclipse extent (in terms of azimuth angle ϕ_s)
λ_s = elevation of the sun in H coordinates (see Appendix)
Ω_p = right ascension of ascending node of planet orbit (see Fig. 3)
Ω_Q = angle between X_E and X_Q (see Appendix), i.e., node of the satellite orbit with respect to the ecliptic (see Fig. 3)
Ω_s = angle between X_P and X_R in the planet orbit plane, i.e., angular distance planet/ecliptic to satellite/planet node (see Fig. 3)
ω_Q = argument of pericenter of the satellite orbit (see Fig. 2)
ρ = solar array rotation, angle between G and Z_B (see Fig. 1)
ρ_1,ρ_2 = two solutions to the ideal alignment condition
θ = satellite yaw angle, i.e., angle between Z_H and Z_B (see Fig. 1)
θ_1,θ_2 = two solutions to the ideal alignment condition
θ_B = best-fit approximation to the ideal alignment (θ_{B1} corresponds to θ_1, and θ_{B2} corresponds to θ_2)
$\Delta\theta_B$ = yaw maneuver amplitude
θ_{BE} = best-fit yaw angle for the eclipse case
θ_{max} = amplitude of the ideal yaw angle profile

Introduction

THE following design task occurs frequently in space technology: a satellite carries a payload that is required to point steadily toward nadir (subsatellite point). Typical examples for such missions are surface observation satellites (carrying remote sensing equipment) or telecommunications satellites with high-gain antennae. The satellite is powered by a plane solar array, e.g., two flat solar panels. The solar array is assumed to have one degree of freedom, namely, its rotation angle ρ. This satellite configuration is shown in Fig. 1.

The task is to keep the payload axis ($-X_B$ in Fig. 1) aligned to the nadir and to control the satellite attitude in such a manner that the solar array normal G points toward the sun in order to maximize the power output. For this purpose, first we have to adjust the solar array rotation angle ρ, which is easily done by a suitable solar array drive and appropriate sunsensors. Second, we have to control a second angle, θ. This is the yaw angle, and it can only be adjusted by controlling the attitude of the spacecraft as a whole.

The first part of this paper deals with the computation of the two attitude angles for the ideal case, i.e., perfect alignment between solar array normal and sun. The problem is solved for the general case of the satellite moving in an elliptical orbit around an arbitrary barycenter. For the sake of brevity, we shall call this barycenter "the planet," but it could be the moon, the Earth, or any other planet in the solar system.

In the second part of the paper, the approximation of the ideal yaw angle profile by a more practical attitude control scheme is proposed.

Derivation of the Ideal Alignment Conditions

For the simple formulation of the mathematical algorithms, the choice of suitable coordinate systems is of great importance. We have chosen two coordinate frames, E and Q, that are centered at the barycenter of the satellite orbit (see Fig. 2) plus two others, B and H (see Figs. 1 and 2, respectively) centered at the satellite. The definition of all coordinates used is given in the Appendix.

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The solar vector S is the unit vector from the barycenter in the direction of the sun. It is defined in E coordinates (see Fig. 2) by Eq. (1):

$$S_E = \begin{bmatrix} \cos s \\ \sin s \\ 0 \end{bmatrix} \tag{1}$$

(*Note:* Since the planets of the solar systems, with the exception of Pluto, all move in orbits close to the ecliptic plane, we can neglect the S_{EZ} component. We further assume that the direction from planet to sun is parallel to the direction from satellite to sun, i.e., both are defined by the same vector S.)

In H coordinates, the solar vector is S_H:

$$S_H = \begin{bmatrix} \cos\alpha\cos(s-\Omega_Q) + \cos i_Q \sin\alpha \sin(s-\Omega_Q) \\ -\sin\alpha\cos(s-\Omega_Q) + \cos i_Q \cos\alpha\sin(s-\Omega_Q) \\ -\sin i_Q \sin(s-\Omega_Q) \end{bmatrix} \tag{2}$$

G is the unit vector perpendicular to the solar array of the satellite. In B coordinates ("body-fixed"), its orientation is defined by the solar array rotation, ρ (see Eq. (3) and Fig. 1):

$$G = \begin{bmatrix} \sin\rho \\ 0 \\ \cos\rho \end{bmatrix} \tag{3}$$

In H coordinates, the solar array vector is G_H:

$$G_H = \begin{bmatrix} \sin\rho \\ -\sin\theta\cos\rho \\ \cos\theta\cos\rho \end{bmatrix} \tag{4}$$

where G_H is thus defined by the yaw angle θ and the solar array rotation ρ. Maximum power is obtained from the solar array if the two vectors S and G are aligned parallel. Comparing Eqs. (2) and (4) give us the ideal alignment conditions, Eqs. (5) and (6):

$$\sin\rho = \cos\alpha\cos(s-\Omega_Q) + \cos i_Q \sin\alpha\sin(s-\Omega_Q) \tag{5}$$

$$\tan\theta = \frac{-\sin\alpha\cos(s-\Omega_Q) + \cos i_Q\cos\alpha\sin(s-\Omega_Q)}{\sin i_Q \sin(s-\Omega_Q)} \tag{6}$$

Let us briefly discuss these alignment conditions and the parameters on which they depend. The parameters i_Q and Ω_Q of the satellite orbit are (in the first approximation) constant. Therefore, the attitude angles ρ,θ are functions of two variables s and α.

The right ascension s of the sun is a well-determined astronomic quantity. When dealing with the Earth as barycenter, s varies through 360 deg in the course of a year and can be determined from the "equation of time." When dealing with other planets, the orbital motion of the planet around the sun determines s. In any case, s is a seasonal variable (long-term variations).

The angle α is a modified true anomaly of the satellite, namely, the sum of the true anomaly ϕ_Q and the argument of pericenter ω_Q:

$$\alpha = \omega_Q + \phi_Q \tag{7}$$

where α is a short-term variable related to the satellite orbit period. To facilitate the discussion of the key formulae, Eqs. (5) and (6), we now introduce two new independent variables. These are the azimuth ϕ_s and the elevation λ_s of the sun, as measured in the H frame (satellite-centered horizontal coordinates). They are defined by the following equation:

$$S_H = \begin{bmatrix} \cos\lambda_s\cos\phi_s \\ \cos\lambda_s\sin\phi_s \\ \sin\lambda_s \end{bmatrix} \tag{8}$$

The relation between these solar angles and the more general quantities defined earlier can be obtained by comparing Eqs. (8) and (2):

$$\tan\phi_s = \frac{-\tan\alpha + \cos i_Q \tan(s-\Omega_Q)}{1 + \tan\alpha\cos i_Q\tan(s-\Omega_Q)} \tag{9}$$

$$\sin\lambda_s = -\sin i_Q \sin(s-\Omega_Q) \tag{10}$$

From Eq. (9) we obtain the following relationship using simple trigonometry and introducing Eq. (7):

$$\phi_s = -\phi_Q - \omega_Q + \tan^{-1}[\cos i_Q \tan(s-\Omega_Q)] \tag{11}$$

Thus, the solar azimuth ϕ_s is a quickly changing parameter that differs from the true anomaly ϕ_Q only by sign and by an additive term; the latter depending on the satellite orbit elements and (i_Q, Ω_Q, and ω_Q) and the sun position (s). Although we shall use the solar azimuth ϕ_s as the short-term independent variable from now on, we could with almost equal ease use the true anomaly ϕ_Q of the satellite.

With the solar angles as defined in Eq. (8) the formulae [Eqs. (5) and (6)] for the attitude angles become extremely simple:

$$\sin\rho = \cos\lambda_s\cos\phi_s \tag{12}$$

$$\tan\theta = -\cot\lambda_s\sin\phi_s \tag{13}$$

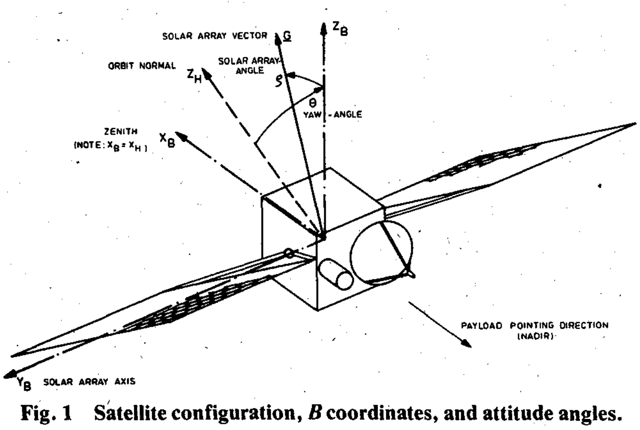

Fig. 1 Satellite configuration, B coordinates, and attitude angles.

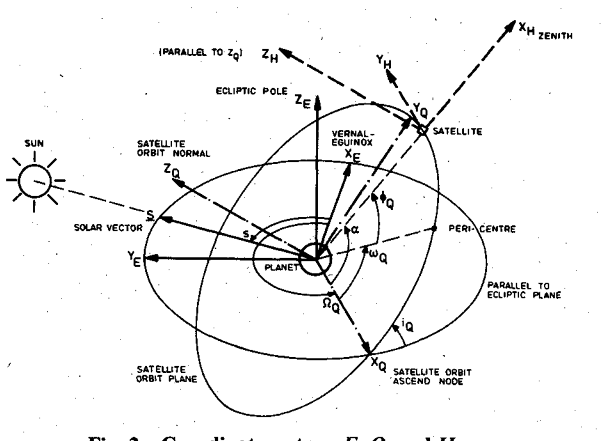

Fig. 2 Coordinate systems E, Q, and H.

From these formulae, the ideal solar array angle ρ is plotted in Fig. 4 and the ideal satellite yaw angle θ in Fig. 5.

We see that both attitude angles are cyclic functions of the solar azimuth and thus of the satellite true anomaly. This is an important and fortunate feature from an engineering point of view. If reaction wheels are used for yaw angle control, the net energy required over one orbit is zero (disregarding small losses). The solar array rotation is limited to less than 360 deg, which is desirable if one wants to avoid slip rings in the solar array drive.

The amplitude of the attitude variations depends only on the solar elevation λ_s. This parameter in turn depends only upon the constant orbit parameters i_Q, Ω_Q, and on the right ascension s of the sun [see Eq. (10)]. Thus, the amplitude is a slowly varying parameter, changing with the season.

Satellite Orbit Related to Planet-Orbit Coordinates

It is common practice to describe the satellite motion in relation to the orbit of the planet around which it moves. The four orbit coordinate systems involved are shown in Fig. 3; the planet orbit is defined by Ω_p, i_p. The satellite orbit is defined in relation to the ecliptic by Ω_Q, i_Q and in relation to the planet orbit by Ω_s, i_s. The definitions are summarized in the Appendix.

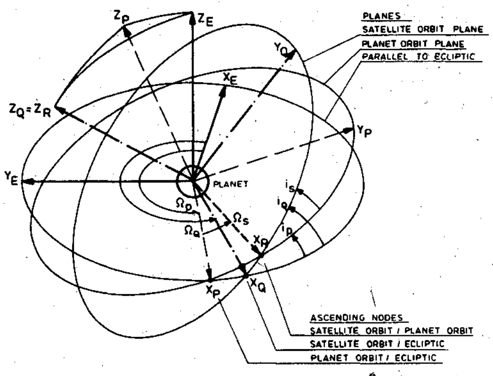

Fig. 3 Coordinate systems *E, P, Q,* and *R*.

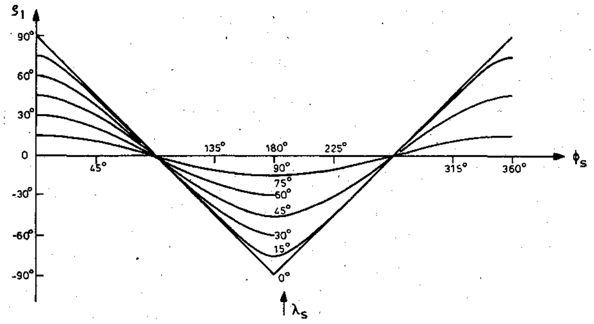

Fig. 4 Ideal solar array rotation ρ_I.

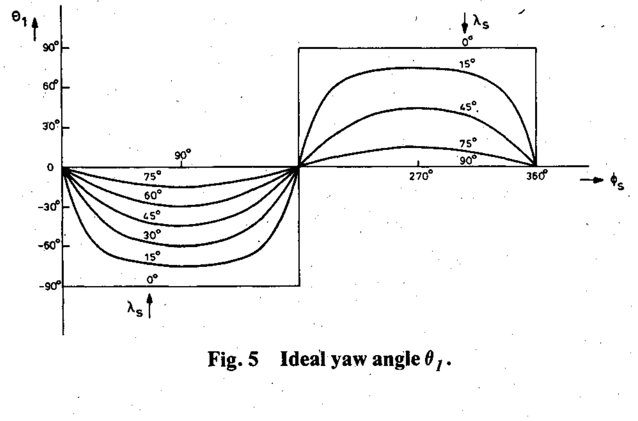

Fig. 5 Ideal yaw angle θ_I.

The orbit parameters in our key formulae, Eqs. (5) and (6), are Ω_Q, i_Q. We now want to express these in terms of the more conventional quantities Ω_p, i_p and Ω_s, i_s. For this purpose, we perform the transformation from the E system to the R system in two ways: 1) by writing down the transformation matrix E to R directly; 2) by doing the process in two steps; namely, transformation E to P followed by transformation P to R. The calculation is straightforward and, therefore, not spelled out here. Comparing the elements of the two matrices obtained in this manner gives us the required relationships:

$$\cos i_Q = -\sin i_p \sin i_s \cos\Omega_s + \cos i_p \cos i_s \qquad (14)$$

$$\sin\Omega_Q = \frac{1}{\sin i_Q}\left[\cos\Omega_p \sin i_s \sin\Omega_s + \sin\Omega_p(\cos i_p \sin i_s \cos\Omega_s + \sin i_p \cos i_s)\right] \qquad (15)$$

$$\cos\Omega_Q = \frac{1}{\sin i_Q}\left[-\sin\Omega_p \sin i_s \sin\Omega_s + \cos\Omega_p(\cos i_p \sin i_s \cos\Omega_s + \sin i_p \cos i_s)\right] \qquad (16)$$

Equations (14-16) are needed for the next paragraph.

Earlier Studies of the General Problem

Dickerson[1] has treated the same problem; however, the coordinates used were the conventional systems (P,S) as described previously. As a consequence, the formulae for the optimum attitude angles would have become unwieldy. Therefore, Dickerson has restricted himself to giving the elements of the transformation matrix for a particular case. The advantage of introducing the Q coordinates in that it allows the writing of formulae, Eqs. (5) and (6), in a manageable form.

The particular case treated in Ref. (1) is that of a satellite orbiting around the moon. The parameters are (in our terminology): $i_s = 90$ deg and $i_p = 5$ deg. Terms smaller than 0.01 are neglected. Equations (14-18) give us, for this case,

$$\cos i_Q = -0.1\cos\Omega_s \qquad (17)$$

$$\Omega_Q = \Omega_p + \Omega_s \qquad (18)$$

With these specific parameters, the ideal attitude angles in Eqs. (5) and (6) assume the following values:

$$\tan\theta = \frac{1}{\sin(s-\Omega_s-\Omega_p)}\left[\sin\alpha\cos(s-\Omega_s-\Omega_p) + 0.1\cos\Omega_s\cos\alpha\sin(s-\Omega_s-\Omega_p)\right] \qquad (19)$$

$$\sin\rho = \cos\alpha\cos(s-\Omega_s-\Omega_p) - 0.1\sin\alpha\cos\Omega_s\sin(s-\Omega_s-\Omega_p) \qquad (20)$$

The results of Ref. 1 can be shown to agree exactly with these formulae; however, the conversion process is laborious and great care is needed in observing signs and definitions.

In a study performed for the European Space Agency, a team of DORNIER[2] has also analyzed this problem for the case of an Earth-orbiting satellite. Again, there is good agreement with the present paper and Ref. 1.

Proposed Attitude Control Scheme

Having discussed the mathematics of the ideal attitude alignment, we now want to give some consideration to the technical implications of the control scheme.

The control of the solar array angle ρ (see Fig. 4) is a simple task. It can be achieved by means of the solar array drive, controlled by a sensor coupled to the solar array. Even a fairly simple system will result in negligible errors, so that in the subsequent discussion, a perfect alignment of ρ is always assumed.

The control of the yaw angle θ (see Fig. 5) is a more demanding task because it requires movement of the complete satellite. This means continuous movement of a body with relatively high moments of inertia (and possibly flexible appendages) according to a fairly complex control law.

We propose, therefore, to use a simplified control scheme as follows. At a true anomaly corresponding to $\phi_s = 0$ deg the yaw angle is adjusted to a value θ_B. The angle $\theta = \theta_B$ is maintained constant until $\phi_s = 180$ deg. At this point, the yaw angle is changed to $\theta = -\theta_B$ and again kept constant up to $\phi_s = 360$ deg. In other words, the function of Fig. 5 is approximated by a rectangular curve. (Actually, the change from $+\theta_B$ to $-\theta_B$ at $\phi_s = 180$ deg can be performed over a finite period, so that the real control curve is trapeze shaped. A look at Fig. 5 shows that this can lead to even better results compared to a truly rectangular curve. However, for analytical purposes the assumption of a rectangular curve is quite sufficient.)

We shall now determine the optimum value for θ_B (the "best-fit yaw angle"). The maximum power output P_0 of the solar array is obtained when the solar array normal G is aligned parallel to the sun vector S. For an arbitrary attitude the output power P is obviously proportional to P_0 and to the cosine of the angle between G and S:

$$P = P_0 G \cdot S \tag{21}$$

This scalar product is calculated from Eqs. (4) and (8). Since we assume perfect adjustment of the solar array rotation ρ, we can, at the same time, introduce the optimum ρ value according to Eq. (12). This gives us

$$P/P_0 = \cos^2\lambda_s\cos^2\phi_s - \cos\lambda_s\sin\theta\sin\phi_s\sqrt{1-\cos^2\lambda_s\cos^2\phi_s} + \sin\lambda_s\cos\theta\sqrt{1-\cos^2\lambda_s\cos^2\phi_s} \tag{22}$$

We now calculate the average power P_m over the half-orbit between $\phi_s = 0$ and $\phi_s = \pi$, assuming the solar elevation λ_s to be approximately constant for the duration of an orbit.

$$P_m = \frac{1}{\pi}\int_0^\pi P(\phi_s, \lambda_s)\,\mathrm{d}\phi_s \tag{23}$$

The integration of the first two terms in Eq. (22) is elementary. The integration of the third term leads to a complete elliptical integral of the second kind, $E(k)$. This function is tabulated in Ref. 3. The modulus k is equal to $\cos\lambda_s$.

$$P_m = \frac{P_0}{2\pi}\left[\pi\cos^2\lambda_s - (\pi - 2\lambda_s + \sin 2\lambda_s)\sin\theta + 4\sin\lambda_s E(\cos\lambda_s)\cos\theta\right] \tag{24}$$

We want to adjust the angle θ to a best-fit value θ_B in such a way that the mean output power P_m becomes a maximum:

$$\frac{\partial P_m}{\partial\theta} = \frac{P_0}{2\pi}\left[-(\pi - 2\lambda_s + \sin\lambda_s)\cos\theta - 4E(\cos\lambda_s)\sin\lambda_s\sin\theta\right] = 0 \tag{25}$$

From this condition, we obtain

$$\tan\theta_B = -\frac{\pi - 2\lambda_s + \sin 2\lambda_s}{4E(\cos\lambda_s)\sin\lambda_s} \tag{26}$$

The best-fit yaw angle θ_B is shown in Fig. 6 as a function of the solar elevation λ_s. As we know from Eq. (10) the parameter λ_s is a function of the (constant) satellite orbit parameters and of the date, as described by the right ascension s of the sun.

Figure 6 shows one particular solution θ_{B1}; a second solution will be discussed in a later paragraph. In addition, Fig. 6 shows the amplitude θ_{1max} of the ideal yaw angle (dotted line).

In order to find out how well the θ_B scheme approximates the ideal yaw alignment (which would produce a solar array output P_0), we have only to introduce the θ_B value from Eq. (26) into Eq. (24) to get the average power $P_m(\theta_B)$. This has been done numerically and the results are shown in Fig. 7 as relative power losses, namely $(P_0 - P_m)/P_0$ in percent.

Figure 7 demonstrates that the scheme works very well. The maximum loss compared to the ideal case occurs at $\lambda_s = 39$ deg and amounts only to 1.80%.

While this penalty is acceptably small, the scheme offers the great advantage that no active yaw control is needed over the largest part of the orbit. This can be particularly important for payloads that have specific viewing requirements with respect to the ground track, e.g., line-scanning experiments that scan in a direction perpendicular to the ground track.

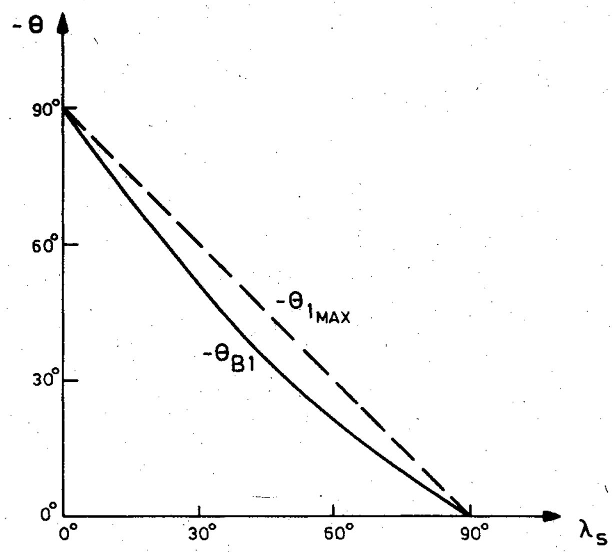

Fig. 6 Best-fit yaw angle θ_{B1}.

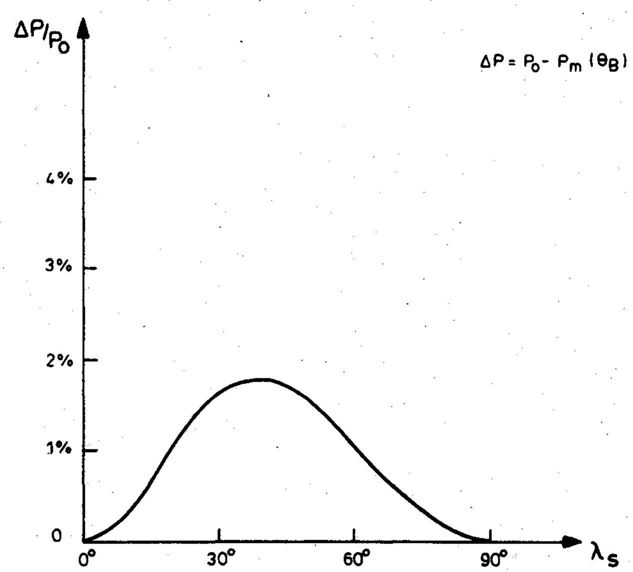

Fig. 7 Power loss for the best-fit attitude.

For this type of experiment the continuous yaw change according to Fig. 5 would require continuous compensation (counterrotation) and would possibly introduce errors (jitter).

Effects of Eclipses

Occultation of the sun by the planet will occur for the parameter combination $\lambda_s = 0$, $\phi_s = 180$ deg and in the vicinity of this point. (See Fig. 2 and Eq. (8); the abovementioned direction is identical with the negative X_H axis.) The solar array power will drop to zero in the region from $\phi_s = \pi - \epsilon_1$ to $\phi_s = \pi + \epsilon_2$. The duration of the eclipse (being $\epsilon_1 + \epsilon_2$) depends on the altitude of the orbit and the size of the planet.

In order to see the effect on the optimum yaw angle, let us look at Fig. 5. In the eclipse region near $\phi_s = 180$ deg no yaw angle requirement is defined. Consequently, the relative influence of the flat part of the yaw profile is increased and the best-fit yaw angle θ_{BE} for the eclipse case lies above the best-fit θ_B for the sunlit orbit. However, the effect is small. For the extreme cases $\lambda_s = 0$ deg and $\lambda_s = 90$ deg, the effect of the eclipse on the yaw optimization is zero, that is, $\theta_{BE} = \theta_B$. For intermediate cases, the angle θ_{BE} lies between θ_B and θ_{max} in Fig. 6.

The value of θ_{BE} can be calculated from Eq. (23) and the subsequent algorithm by adjusting the integration limits. The following arbitrary example illustrates the magnitude of the effect. For a sun elevation $\lambda_s = 45$ deg and a symmetrical eclipse $\epsilon_1 = \epsilon_2 = 45$ deg, we obtain an optimum yaw angle of $\theta_{BE} = 42.03$ deg. For a sunlit orbit with $\lambda_s = 45$ deg, we would get an optimum angle of $\theta_B = 34.14$ deg.

Second Set of Optimum Attitude Angles

Let us return to Eqs. (12) and (13) for the ideal attitude angles. Obviously Eq. (12) is satisfied by two different values, ρ_1 and ρ_2.

$$\rho_2 = \pi - \rho_1 \tag{27}$$

In order to find the quadrant of the corresponding yaw angle θ_2, we use Eqs. (4) and (8) because Eq. (13) does not define the quadrant:

$$\sin\theta = -\cos\lambda_s \sin\phi_s / \cos\rho \tag{28}$$

$$\cos\theta = \sin\lambda_s / \cos\rho \tag{29}$$

With Eq. (27) we obtain the second yaw angle as

$$\theta_2 = \pi + \theta_1 \tag{30}$$

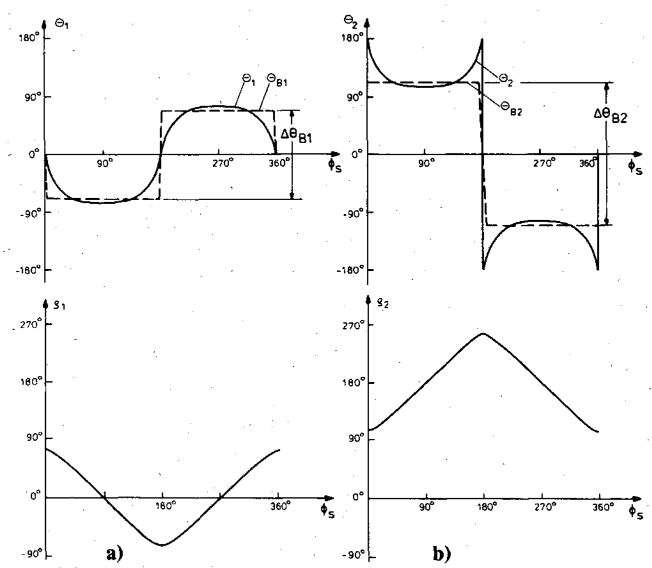

Fig. 8 Two sets of ideal angles θ, ρ (examples for $\lambda_s = 15$ deg): a) set θ_1, ρ_1; b) set θ_2, ρ_2.

The physical significance of the two solutions is easily seen. The rotation axis Y_B of the solar array must be perpendicular to both the sun vector S and the local vertical X_H. That means the orientation of the Y_B axis in space is determined, except for its sign. Hence, two yaw angles, θ_1 and θ_2, differing by 180 deg, will fulfill the optimization conditions.

It should be noted that both sets of attitude angles result in a parallel (not antiparallel) alignment of vectors G and S; the scheme does not, therefore, require solar cells on both sides of the array.

The two different sets of ideal attitude angles are shown in Figs. 8a and 8b (for an arbitrary example in which the parameter $\lambda_s = 15$ deg was selected). In addition Fig. 8 also shows the best-fit yaw angles θ_{B1} and θ_{B2} from Eq. (26).

The existence of a second set of optimum attitude angles leads to a practical advantage in the control operations; namely, the reduction of yaw maneuver amplitudes. As explained earlier, our proposal is to approximate the ideal yaw angle curve (Fig. 5) by a rectangular curve. This implies a yaw maneuver at two points in the orbit ($\phi_s \mp 0$ and 180 deg).

We call the amplitude of this maneuver $\Delta\theta_B$. For the first solution the amplitude $\Delta\theta_{B1}$ can be seen in Fig. 8a. Its value is

$$\Delta\theta_{B1} = -2\theta_{B1} \tag{31}$$

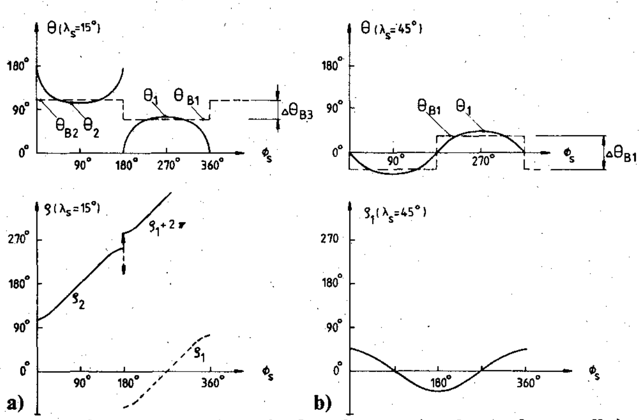

Fig. 9 Most economic attitude scheme: a) scheme for small λ_s; b) scheme for large λ_s.

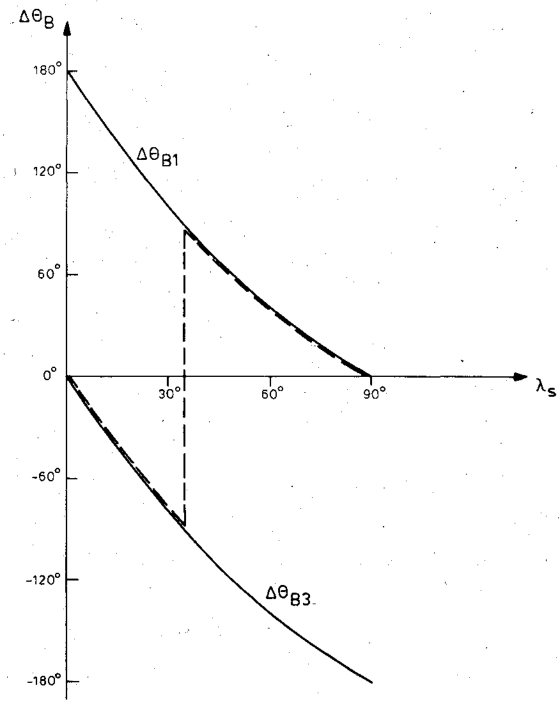

Fig. 10 Yaw angle amplitude $\Delta\theta_B$.

For the second solution, the amplitude according to Fig. 8b and Eq. (30) is:

$$\Delta\theta_{B2} = -2(\pi + \theta_{B1}) \qquad (32)$$

The best-fit angle θ_{B1} is shown in Fig. 6. We see that very large amplitudes are required at small sun elevations λ_s, the worst case being an amplitude of 180 deg at zero sun elevation. For the second solution (see Fig. 8b), equally large amplitudes are required.

Such large slew maneuvers may be very inconvenient in practice. Therefore, a scheme is proposed that halves the requried maximum amplitude. This scheme is illustrated in Fig. 9.

At small sun elevations λ_s (see Fig. 9a), we start the orbit cycle at $\phi_s = 0$ deg with solution θ_{B2}. When we get to $\phi_s = 180$ deg we switch to solution ϕ_{B1}. The maneuver required at this point has an amplitude which we call $\Delta\theta_{B3}$. This amplitude can easily be shown to have the following value:

$$\Delta\theta_{B3} = \Delta\theta_{B1} - \pi \qquad (33)$$

The absolute value $|\Delta\theta_{B3}|$ is smaller than $|\Delta\theta_{B1}|$ for the parameter range 0 deg $< \lambda_s < 35$ deg. For sun elevations $\lambda_s > 35$ deg we choose solution 1, requiring a maneuver amplitude $\Delta\theta_{B1}$ (see Fig. 9b).

The yaw amplitudes $\Delta\theta_{B1}$ and $\Delta\theta_{B3}$ are plotted in Fig. 10 as a function of the solar elevation λ_s. The "most economic yaw scheme," which is described above, is indicated in Fig. 10 by a dotted line. We see that this scheme limits the absolute value of the required yaw amplitude to 90 deg.

One remark concerning the solar array rotation ρ in Fig. 9a ought to be added. In the scheme for small elevations λ_s, the angle undergoes a step change at $\phi_s = 180$ deg. This is a fairly large step in the negative direction. For solar array drives that have no limited ranges (i.e., those equipped with slip rings), the amplitude of this step can be reduced by jumping from ρ_2 to $\rho_1 + 2\pi$ (solid line) instead of from ρ_2 to ρ_1 (dotted line).

Conclusions

The type of satellite discussed herein requires an adjustment of the yaw attitude as a function of the satellite's true anomaly. The ideal yaw angle profile can be approximated by a square-wave function, which is technically simpler to realize. In addition, it benefits certain types of payload, since the pointing accuracy is not disturbed. The power loss of this scheme as compared to the case of ideal alignment is remarkably small, having a maximum value of 1.8%.

A further practical advantage can be derived from the fact that the alignment condition is satisfied by two sets of attitude angles. This can be used to limit the yaw amplitude to values not exceeding 90 deg for any elevation of the sun.

Appendix—Definition of Coordinate Systems

E system: "Ecliptic" coordinates centered at the barycenter of satellite orbit (see Figs. 2 and 3).

- X_E = axis toward vernal equinox, parallel to ecliptic plane
- Z_E = normal to ecliptic plane
- Y_E = completed right-handed tripod

P system: "Planet-orbit" coordinates centered at the barycenter of satellite orbit (see Fig. 3).

- X_p = axis in the planet-orbit plane toward ascending node with the $X_E - Y_E$ plane (ecliptic)
- Z_p = normal to the planet orbit plane, direction of orbit rate vector
- Y_p = completed right-handed tripod

Q system: "Satellite-orbit" coordinates, centered at the barycenter of satellite orbit (see Figs. 2 and 3).

- X_Q = axis in the satellite orbit plane toward ascending node with the $X_E - Y_E$ plane (ecliptic)
- Z_Q = normal to the satellite orbit plane, direction of orbit rate vector
- Y_Q = completed right-handed tripod

R system: Same as Q system, except for the definiton of X_R

- X_R = axis in the satellite orbit plane toward ascending node with the $X_p - Y_p$ plane (planet orbit plane)
- Z_R = Z_Q = normal to the satellite orbit, direction of orbit rate vector
- Y_R = completed right-handed tripod

B system: "Body fixed" coordinates, satellite centered (see Fig. 1).

- X_B = opposite to payload viewing direction, i.e., toward zenith
- Y_B = rotation axis of solar array
- Z_B = completed right-handed tripod

H system: "Horizontal" coordinates, satellite centered (see Fig. 2).

- X_H = axis toward zenith
- Y_H = axis parallel to orbital velocity vector
- Z_H = satellite orbit normal

References

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